$$= -\frac{k\nu^2}{2c_pL^3}\left(\frac{c_fh}{2}\right)^{1/2}\left\{g_\infty + \frac{4}{a_0}\left[\frac{1}{2}\left(a_0^2 - \frac{g_0}{4}\right)\times \left(-\frac{1}{\pi}+\frac{1}{2}\right) + \frac{g_\infty a_1}{2a_0^{1/2}}\left(\frac{1}{\pi^{1/2}} - \frac{\pi^{1/2}}{4}\right)\right]\xi^2 + \ldots\right\}$$

$$Pr = 1 \quad (31)$$

References

[1] Schaaf, S. A. and Sherman, F. S., "Skin friction in slip flow," J. Aeronaut. Sci. 21, 85–90, 144 (1954).

[2] Hasimoto, H., "Boundary layer slip solutions for a flat plate," J. Aeronaut. Sci. 25, 68–69 (1958).

[3] Hassan, H. A., "On skin friction in the slip-flow regime," J. Aeronaut. Sci. 28, 335–336 (1961).

[4] Street, R. E., "A study of boundary conditions in slip-flow aerodynamics," *Rarefied Gas Dynamics* (Pergamon Press, Inc., New York, 1960), pp. 276–292.

[5] Sparrow, E. M. and Lin, S. H., "Laminar heat transfer in tubes under slip-flow conditions," Trans. Am. Soc. Mech. Engrs. Ser. C: J. Heat Transfer 84, 363–369 (November 1962).

Pressure Distribution for Hypersonic Boundary-Layer Flow

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A pressure distribution is derived here, which is in excellent agreement with the tangent-wedge approximation.

Introduction

In solving the hypersonic boundary-layer momentum integral equation [Eq. (2.12) of Ref. 1], it is necessary to find out the pressure distribution $P^*(= P_2/P_1$, where subscript 2 stands for $x = x$, $y = \delta$, and subscript 1 stands for $x = \infty$, $y = \delta$; x, y are the surface and normal-to-surface coordinates, and δ is the boundary-layer thickness) in terms of η and ξ, where $\eta = \delta/L$ and $\xi = x/L$ (where L equals a certain characteristic length defined in Ref. 1). Here, one such pressure distribution is derived from the fundamental shock-relations.

Shock Relations and Derivation of Pressure Distribution

From shock relations,

$$P_2/P_{-\infty} = [2\gamma/(\gamma+1)]M_{-\infty}^2\sin^2\theta - [(\gamma-1)/(\gamma+1)] \quad (1)$$

where γ is the specific heats ratio; $M_{-\infty}$ is the freestream Mach number; θ is the shock angle; and $P_{-\infty}$ is the freestream pressure.

Equation (1) also can be written as

$$(P_2 - P_{-\infty})/P_{-\infty} = (2\gamma/\gamma+1)(M_{-\infty}^2\sin^2\theta - 1) \quad (2)$$

Also, shock angle and deflection angle relation is

$$M_{-\infty}^2\sin^2\theta - 1 = \frac{\gamma+1}{2}\,\frac{M_{-\infty}^2\sin\theta\sin\Delta}{\cos(\theta-\Delta)} \quad (3)$$

where Δ is the deflection angle.

Received May 13, 1963. My thanks are due to J. N. Kapur, Professor and Head of Mathematics Department, Indian Institute of Technology, Kanpur for his advice in the preparation of the present paper.

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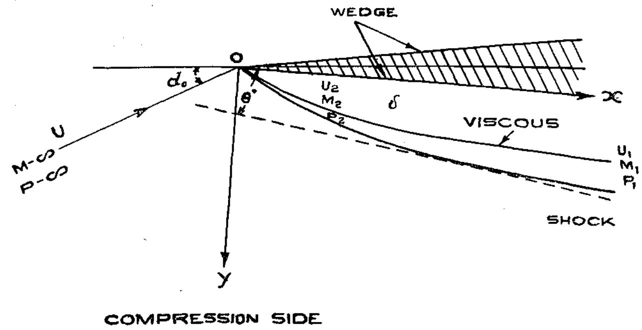

Fig. 1 Hypersonic viscous flow over an insulated wedge at an angle of attack.

Now for slender wedges, $\cos(\theta - \Delta) \approx 1$ and $\sin\Delta \approx \Delta$. Therefore (3) becomes

$$M_{-\infty}^2\sin^2\theta - 1 = [(\gamma+1)/2]M_{-\infty}^2\sin\theta\cdot\Delta \quad (4)$$

Hence,

$$\frac{P_2 - P_{-\infty}}{P_{-\infty}} = \frac{2\gamma}{\gamma+1}\left[\frac{\gamma+1}{2}M_{-\infty}^2\sin\theta\right]\Delta$$

$$= \gamma M_{-\infty}^2\sin\theta\cdot\Delta \quad (5)$$

Now as in Ref. 1 in order to satisfy the asymptotic conditions at $x = +\infty$, let $\theta = \theta_0 + \theta_1$ and $\Delta = \Delta_0 + \Delta_1$, where θ_0 and Δ_0 are the inviscid shock and deflection angles, and θ_1 and Δ_1 correspond to viscous shock and deflection angles. So, (5) becomes

$$P_2/P_{-\infty} = 1 + \{\gamma M_{-\infty}^2\sin(\theta_0+\theta_1)\}(\Delta_0+\Delta_1)$$
$$= 1 + \gamma M_{-\infty}^2(\Delta_0+\Delta_1)(\sin\theta_0 + \theta_1\cos\theta_0)$$

where $\cos\theta_1 \approx 1$ and $\sin\theta_1 \approx \theta_1$. Therefore,

$$P_2/P_{-\infty} = 1 + \gamma M_{-\infty}^2\sin\theta_0\cdot\Delta_0 + \gamma M_{-\infty}^2\sin\theta_0\cdot\Delta_1 + \gamma M_{-\infty}^2\cos\theta_0\cdot\Delta_0\theta_1 + \gamma M_{-\infty}^2\cos\theta_0\cdot\Delta_1\theta_1 \quad (6a)$$

i.e.,

$$P_2/P_{-\infty} = P_{+\infty}/P_{-\infty} + \gamma M_{-\infty}^2\Delta_1\cdot\sin\theta_0 + \gamma M_{-\infty}^2\cos\theta_0\cdot\theta_1\Delta_0 + \gamma M_{-\infty}^2\cos\theta_0\theta_1\Delta_1 \quad (6b)$$

where $P_{+\infty}/P_{-\infty} = P_1/P_{-\infty}$ = pressure ratio across the shock for the corresponding inviscid flow.

A. Wedge case

As in Ref. 1, for the case of wedge,

$$\theta_1 \approx [(\gamma+1)/4]\Delta_1 \quad (7)$$

when θ_0 is also very much less than 1 rad, and $\theta_0 > \theta_1$.

Approximating (6b) in view with (7),

$$\frac{P_2}{P_{+\infty}} = \frac{P_2}{P_1} = 1 + \gamma M_{-\infty}^2\sin\theta_0\Delta_1\cdot\frac{P_{-\infty}}{P_{+\infty}} + \gamma M_{-\infty}^2\cos\theta_0\frac{P_{-\infty}}{P_{+\infty}}\theta_1\Delta_0 + \gamma M_{-\infty}^2\cos\theta_0\frac{P_{-\infty}}{P_{+\infty}}\Delta_1\theta_1$$

i.e.,

$$P^* = 1 + \frac{P_{-\infty}}{P_{+\infty}}\gamma M_{-\infty}^2\sin\theta_0\cdot\Delta_1 + \frac{P_{-\infty}}{P_{+\infty}}\gamma M_{-\infty}^2\cos\theta_0\cdot\Delta_0\frac{\gamma+1}{4}\Delta_1 + \gamma M_{-\infty}^2\frac{P_{-\infty}}{P_{+\infty}}\cos\theta_0\cdot\frac{\gamma+1}{4}\Delta_1^2$$

but

$$\Delta_1 \approx d\eta/d\xi \quad (8)$$

Therefore,

$$P^* = 1 + \frac{P_{-\infty}}{P_{+\infty}} \gamma M_{-\infty}^2 \left(\sin\theta_0 + \frac{\gamma + 1}{4} \cos\theta_0 \cdot \Delta_0\right) \frac{d\eta}{d\xi} + \frac{P_{-\infty}}{P_{+\infty}} \gamma M_{-\infty}^2 \cos\theta_0 \cdot \frac{\gamma + 1}{4} \left(\frac{d\eta}{d\xi}\right)^2$$

$$= 1 + B\left(\frac{d\eta}{d\xi}\right) + C\left(\frac{d\eta}{d\xi}\right)^2 \quad (9)$$

where

$$B = (P_{-\infty}/P_{+\infty})\gamma M_{-\infty}^2(\sin\theta_0 + [(\gamma + 1)/4] \cos\theta_0 \cdot \Delta_0) \quad (10a)$$

and

$$C = (P_{-\infty}/P_{+\infty})\gamma M_{-\infty}^2 \cos\theta_0 \cdot [(\gamma + 1)/4] \quad (10b)$$

B. Special case of flat plate placed along direction of freestream

Here

$$\alpha = \theta_0 = \sin^{-1}1/M_{-\infty} = \sin^{-1}1/M_1$$

$$\cos\theta_0 \approx 1$$

for which $P_{+\infty}/P_{-\infty} = 1$ and $\Delta_0 = 0$. Equation (6b) becomes

$$P^* = 1 + \gamma M_1^2(1/M_1)\Delta_1 + \gamma M_1^2\theta_1\Delta_1$$

$$= 1 + \gamma M_1\Delta_1 + \gamma M_1^2\theta_1\Delta_1 \quad (11)$$

From Ref. 1, for the strong interaction case,

$$\theta_1 \approx [(\gamma + 1)/2]\Delta_1 \quad (12)$$

and, for weak interaction case,

$$\theta_1 \approx [(\gamma + 1)/4]\Delta_1 \quad (13)$$

Corresponding to (12), P^* is given from (11) as

$$P^* = 1 + \gamma M_1 \frac{d\eta}{d\xi} + \gamma M_1^2 \cdot \frac{(\gamma + 1)}{2} \left(\frac{d\eta}{d\xi}\right)^2 \quad \left(\text{since } \Delta_1 \approx \frac{d\eta}{d\xi}\right)$$

$$= 1 + \gamma k + \gamma \frac{(\gamma + 1)}{2} k^2 \quad (14)$$

where k is hypersonic similarily parameter $k = M_1(d\eta/d\xi)$. For (13), P^* is

$$P^* = 1 + \gamma k + \gamma[(\gamma + 1)/4]k^2 \quad (15)$$

Conclusion

In (14), i.e., for strong interaction case, if $(1 + \gamma k)$ is neglected in comparison to $[\gamma(\gamma + 1)/2]k^2$, since for this case $k \gg 2$, P^* becomes

$$P^* \approx [\gamma(\gamma + 1)/2]k^2$$

which is exactly the distribution represented by the tangent wedge.

Equation (15) contains the first three terms of the series for P^*, given by tne tangent wedge for the weak interaction case. Also when for weak interaction $P^* = 1 + \gamma k$ and for strong interaction $P^* \approx [\gamma(\gamma + 1)/2]k^2$, the pressure distribution for the whole flow system is often represented by $1 + \gamma k + \gamma[(\gamma + 1)/2]k^2$.

From (14) and (15), the forementioned case easily is seen to be

$$P^* = 1 + \gamma k + [\gamma(\gamma + 1)/4]k^2 + [\gamma(\gamma + 1)/2]k^2$$

If $[\gamma(\gamma + 1)/4]k^2$ is neglected, since for weak interaction case $k \ll 2$,

$$P^* = 1 + \gamma k + [\gamma(\gamma + 1)/2]k^2$$

which is the required distribution.

From the forementioned relations of (14) and (15), it is easy to see that the pressure distribution (6b) is in excellent agreement with the tangent-wedge approximation.

Reference

[1] Pai, S., "Hypersonic viscous flow over an insulated wedge at an angle of attack," Univ. Md. Rept. B.N. 42, Air Res. Dev. Command, Office Sci. Res. Rept. OSR-TN-54-321 (October 1954).

Capture of a Passively Stabilized Satellite by Earth's Gravity Field

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INTEREST is currently high in passive stabilization of spacecraft, particularly satellites of the earth. Of the several techniques of passive stabilization, use of earth's gravity field is receiving perhaps most attention. In this technique, the satellite's configuration is so arranged that torques are produced on the satellite by the gradient of earth's gravity field, restoring the satellite toward one of its equilibrium positions if displaced from equilibrium. Under the action of these torques, the satellite librates or oscillates about one of the equilibrium positions. If the satellite also contains some damping mechanism, the oscillations will decay until they fall within a small steady-state deadband about one of the equilibrium positions. The satellite is so configured that in its equilibrium position an antenna, for instance, is directed to earth.

The classic configuration that is torqued by earth's gravity gradient is a dumbbell. The dumbbell is oriented in pitch and roll but not yaw, where the yaw axis is the local vertical. Another configuration, one now being widely studied and developed, consists of a central body from which long thin-walled tubes uncoil once the satellite is in orbit. Because of their great length (hundreds of feet for some applications), the tubes endow the satellite with the necessary inertia to develop from the gravity field high restoring torques. If the tubes extend from the central body to form a cross having its two members at right angles and of unequal length, restoring torques in yaw, as well as pitch and roll, are developed. Yaw restoring torques are also developed if the rods extend to form an X, that is, with two members of equal length intersecting at an angle different from 90°.

A typical mission profile of such a gravity oriented satellite consists of the following:

1) Injection into orbit from a final propulsion stage either spin-stabilized such as Scout or containing its own attitude stabilization such as Agena. In the former instance the satellite, after separation from the propulsion stage, is itself spinning. In the latter instance the satellite is slowly tumbling after separation.

2) Conversion of the spinning or tumbling of the satellite to oscillation about the local vertical. Oscillation about the local vertical, when seen from inertial space, is a steady rotation of the satellite plus a superimposed oscillation. The rotation is about an axis normal to the orbital plane. The rotation is at a rate eoual to the satellite's orbital rate, which is much slower than the rate at injection. As seen from inertial space, then, the spinning or random tumbling at injection must be converted to a much slower rotation about an axis having a particular direction in space.

3) Decay of the satellite's oscillation. The oscillation decays to a small deadband whose width is determined by the

Received May 16, 1963; revision received August 2, 1963.

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